285 mb at the same nucleon energy. However, not all of the deuteron total cross section contributes in our geometry, since some of the stripped protons continue on through our telescope. Using the results of Franco,[12] we estimate that the mean free path for deuterons in CO_2 is 0.5 ± 0.1 times the mean free path for protons in the same gas for the 11.6-BeV/c measurement. For this measurement the combined absorption factor for all material in the beam is 4.1. The over-all correction factor for the 11.6-BeV/c measurement which includes the efficiency of the gas Čerenkov for deuterons of this velocity comes to 10.8.

The final cross-section values, given in Table I, have an estimated overall uncertainty of $+100\%$, -50%.

Differential cross sections in the region 10^{-34} to 10^{-35} cm^2/sr in the c.m. system were obtained, which represent a decrease over previous experimental values by a factor of $\sim 10^4$. The differential cross section is strongly dependent on the c.m. scattering angle, and upon the total energy involved in the interaction.

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Sum Rules for Coupling Constants in Broken SU(3) Symmetry

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The coupling constant sum rules for the decay of an octet, icosuplet and 27-plet to the two octets are derived in the broken SU(3) symmetry scheme.

I. INTRODUCTION

We have recently given sum rules for the coupling constants of the unitary decuplet to two unitary octets for the broken SU(3) symmetry.[1] As one of these sum rules was entirely in terms of the observed decay widths of $N^*(1238)\to N\pi$, $Y_1^*(1385)\to\Lambda\pi$ and $\Sigma\pi$, $\Xi^*(1530)\to\Xi\pi$, it could be checked against experimental data. The agreement was very good. Apart from qualitative predictions[2] about multiplet structure and spin-parity assignments, this result and the mass sum rules[3] are the only quantitative results in the broken SU(3) symmetry[4] which have had experimental confirmation. This makes one have more confidence in SU(3) symmetry with the usual breaking term, i.e., the one having transformation properties of the $I=0$, $Y=0$ component of a unitary octet. As in the domain of coupling constant sum rules, there are further obvious possibilities of deriving useful results, we extend our previous work here. Since, experimentally, the most interesting case is the one in which, out of three multiplets involved, the two are octets. We only consider this case in the present paper.

The following cases of interest arise:

(a) $B^*(8)\to B(8)+M(8)$, where B^*, B, and M refer to meson-baryon resonance; baryon and meson octets, respectively.

(b) Icosuplet $\to$ octet$+$octet, where the two octets are different in general.

(c) $B^*(27)\to B(8)+M(8)$ and also the case of a meson 27-plet, $M(27)\to M(8)+M'(8)$.

In all the calculations we treat the $I=0$, $Y=0$ breaking term to first order only. Further, we use the "spurion octet" technique (outlined in Ref. 1) and de Swart's tables[5] for the SU(3) Clebsch-Gordan coefficients throughout the paper.

II. SUM RULES FOR $B^*(8)\to B(8)+P(8)$

The coupling-constant sum rules for the particular case when the two of the three octets are the same have already been given by Glashow and Muraskin.[6] We generalize their results to the case when all the three

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octets are different. For definiteness, we take the octets to be $B^*(8)$, $B(8)$, and $P(8)$. The isospin multiplets of B^* are denoted by N^*, Y_0^*, Y_1^*, and Ξ^* in an obvious notation. This case is of practical importance since two possible meson-baryon resonance octets are indicated by experiment. These are

(i) A $J^P=\frac{3}{2}^-$ octet with $N^*(1512)$ and $Y_0^*(1520)$ known[7] with Y_1^* and Ξ^* missing. To get an idea regarding their masses one can assume $[m(\Sigma)-m(\Lambda)]\sim[m(Y_1^*)-m(Y_0^*)]$ and this gives in $(Y_1^*)\sim 1600$ MeV and $m(\Xi^*)\sim 1568$ MeV. These values seem unlikely, though, as this energy range has been experimentally investigated. On the other hand, assuming $[m(\Xi)-m(N)]\sim[m(\Xi^*)-m(N^*)]$ one expects $m(Y_1^*)\sim 2250$ MeV and $m(\Xi^*)\sim 1900$ MeV.

(ii) A $J^P=\frac{5}{2}^+$ octet with $N^*(1688)$ and $Y_0^*(1815)$ known[7] and Y_1^* and Ξ^* missing as yet. Similarly, one can estimate the masses, for this case, as for the $\frac{3}{2}^-$ octet. Assuming $[m(\Sigma)-m(\Lambda)]\sim[m(Y_1^*)-m(Y_0^*)]$ and $[m(\Xi)-m(N)]\sim[m(\Xi^*)-m(N^*)]$ one expects the missing Y_1^* and Ξ^* to lie at ~ 1900 and ~ 2000 MeV, respectively.

III. SUM RULES

In the general case when all the three octets are different there are seventeen coupling constants and ten parameters, two from exact symmetry and eight from first-order symmetry breaking. Thus, we expect seven sum rules, viz.,

$$2X(Y_1^*,N\bar{K})+2X(Y_1^*,\Sigma\pi)+2X(\Xi^*,\Xi\pi) = 2X(Y_1^*,\Lambda\pi)+X(\Xi^*,\Sigma\bar{K})+X(\Xi^*,\Lambda\bar{K}), \quad (3.1)$$

$$2X(Y_1^*,N\bar{K})+2X(Y_1^*,\Sigma\pi)+2X(N^*,\Sigma K) = 2X(Y_1^*,\Sigma\eta)+X(N^*,N\pi)+X(N^*,N\eta), \quad (3.2)$$

$$2X(Y_1^*,\Sigma\pi)+2X(N^*,\Sigma K)+2X(\Xi^*,\Xi\pi) = 2X(Y_0^*,\Sigma\pi)+X(Y_1^*,\Xi K)+X(Y_0^*,\Xi K), \quad (3.3)$$

$$2X(Y_1^*,\Xi K)+2X(N^*,N\pi) = 2X(Y_1^*,\Sigma\pi)+2X(Y_1^*,\Lambda\pi) +X(N^*,\Sigma K)+X(N^*,\Lambda K), \quad (3.4)$$

$$2X(Y_1^*,\Xi K)+2X(\Xi^*,\Sigma\bar{K}) = 2X(Y_1^*,\Sigma\pi)+2X(Y_1^*,\Sigma\eta) +X(\Xi^*,\Xi\pi)+X(\Xi^*,\Xi\eta), \quad (3.5)$$

$$2X(N^*,N\pi)+2X(\Xi^*,\Sigma\bar{K}) = 2X(Y_1^*,\Sigma\pi)+2X(Y_0^*,\Sigma\pi) +X(Y_1^*,N\bar{K})+X(Y_0^*,N\bar{K}), \quad (3.6)$$

$$3X(Y_0^*,\Lambda\eta)+3X(Y_1^*,\Sigma\eta)+3X(Y_1^*,\Lambda\pi) = 3X(Y_1^*,N\bar{K})+3X(Y_1^*,\Xi K)+X(Y_0^*,N\bar{K}) +X(Y_0^*,\Xi K)+X(Y_0^*,\Sigma\pi). \quad (3.7)$$

Though these sum rules have been given in terms for $B^*(8)\rightarrow B(8)+P(8)$, they clearly apply to the case of any three octets with the appropriate changes in the particle labels.

The actual coupling constants G's are given by $G=$(algebraic factor from exact symmetry for D coupling). X, except for $G(Y_1^*,\Sigma\pi)$ which is equal to $6^{1/2}/5^{1/2}X(Y_1^*,\Sigma\pi)$. For example,

$$G(Y_0^*,N\bar{K})=1/10^{1/2}X(Y_0^*,N\bar{K}).$$

The width for a decay mode is given by

$$\Gamma=|G|^2p^{2l+1}m_B/m_{B^*},$$

where m_{B^*} is the resonance mass, m_B the baryon mass, and "p" the final center-of-mass momentum.

The Glashow and Muraskin[6] sum rules for $B^*\equiv B$ can be obtained by putting

$$X(Y_1^*,N\bar{K})=X(N^*,\Sigma K),\quad X(Y_0^*,N\bar{K})=X(N^*,\Lambda K),$$
$$X(Y_1^*,\Xi K)=X(\Xi^*,\Sigma\bar{K}),\quad X(Y_0^*,\Xi K)=X(\Xi^*,\Lambda\bar{K}),$$
$$X(Y_1^*,\Lambda\pi)=X(Y_0^*,\Sigma\pi).$$

Then the sum rules (3.4) and (3.6) become the same and (3.1) and (3.3) become the same thus giving only five independent sum rules as expected.

IV. DISCUSSION OF SUM RULES

All the seven simple sum rules in Sec. III above involve the decays of Y_1^*, while (3.7) involves Y_1^* and Y_0^* coupling constants only. Since we know the masses of the N^* and Y_0^* in the $\frac{3}{2}^-$ and $\frac{5}{2}^+$ octets it is clear that the rules 2, 3, 4, and 7 in Sec. III cannot be easily tested as they involve $G(N^*,\Sigma K)$ and $G(Y_0^*,\Xi K)$. However, the rules (3.5) and (3.6) are more hopeful. In fact, if the missing Y_1^* and Ξ^* both lie at or above 1900 MeV then the rules (3.1), (3.5), and (3.6) may be amenable to experimental verification. Of these the rules (3.1) and (3.6) would be easier to verify than (3.5).

Note added in proof. The $3/2^-$ octet has been identified as: $N^*(1512)$, $Y_1^*(1660)$, $Y_0^*(1650)$, and $\Xi^*(1820)$, with $Y_0^*(1520)$ presumably being a $3/2^-$ unitary singlet. Consequently, the sum rules (3.1) and (3.6) can be checked by experiment. We are grateful to Professor Y. Ne'eman for the experimental information.

V. SUM RULES FOR ICOSUPLET COUPLING CONSTANTS

It has been suggested[8] that the recently observed boson resonances between the pseudoscalar octet P and the vector meson octet $V(8)$ may be classified according to a self-charge conjugate icosuplet.

A. Boson Self-Charge Conjugate Icosuplet

The two different isotriplets with $Y=0$, one in the representation 10 and the other in 10*, are not eigen-

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states of G conjugation. The observable isotriplets would, however, be states with definite G parity, and would be linear combinations of the former. Let B_1 and B_2 stand for $I=1$, $Y=0$ states with $G=+1$ and $G=-1$, respectively. Then

$$G(B_1,K^*\bar{K})=G(B_1,\rho\pi)=G(B_1,\bar{K}^*K)=0$$

and

$$G(B_2,\omega\pi)=G(B_2,\rho\eta)=0\,,$$

and hence these decays will be forbidden.

Let $M_{I,Y}$ stand for the (I,Y) component of the icosuplet. Then the sum rules for this case can be obtained from those previously given by the present authors for the decuplet,[1]

$$X(N^*,N\pi)\rightarrow X(M_{3/2,1},K^*\pi)\,, \qquad (5.1)$$

$$X(N^*,\Sigma K)\rightarrow X(M_{3/2,1},\rho K)\,, \qquad (5.2)$$

$$\sqrt{2}X(Y_1^*,\Lambda\pi)\rightarrow X(B_1,\omega\pi)\,, \qquad (5.3)$$

$$\sqrt{2}X(Y_1^*,\Sigma\eta)\rightarrow X(B_1,\rho\eta)\,, \qquad (5.4)$$

$$-\sqrt{2}X(Y_1^*,N\bar{K})\rightarrow X(B_2,K^*\bar{K})\,, \qquad (5.5)$$

$$-\sqrt{2}X(Y_1^*,\Xi K)\rightarrow X(B_2,\bar{K}^*K)\,, \qquad (5.6)$$

$$-\sqrt{2}X(Y_1^*,\Sigma\pi)\rightarrow X(B_2,\rho\pi)\,, \qquad (5.7)$$

$$X(\Xi^*,\Lambda\bar{K})\rightarrow X(M_{1/2,-1},\omega\bar{K})\,, \qquad (5.8)$$

$$X(\Xi^*,\Sigma\bar{K})\rightarrow X(M_{1/2,-1},\rho\bar{K})\,, \qquad (5.9)$$

$$X(\Xi^*,\Xi\eta)\rightarrow X(M_{1/2,-1},\bar{K}^*\eta)\,, \qquad (5.10)$$

$$X(\Xi^*,\Xi\pi)\rightarrow X(M_{1/2,-1},\bar{K}^*\pi)\,, \qquad (5.11)$$

$$X(\Omega^-,\Xi\bar{K})\rightarrow X(M_{0,-2},\bar{K}^*\bar{K})\,. \qquad (5.12)$$

In the present case, of course, we also have

$$X(B_2,K^*\bar{K})=X(B_2,\bar{K}^*K)\,. \qquad (5.13)$$

The charge-conjugate decays obviously will have the corresponding X's equal.

B. Fermion Icosuplet with R Symmetry

In this case again the two isotriplets with $Y=0$ will mix, so as to become eigenstates of R conjugation. Let $Y_1^{(1)}$ and $Y_1^{(2)}$ stand for R-symmetric and R-antisymmetric states.

The sum rules for this case, i.e., the fermion icosuplet decaying into the baryon octet (N,Σ,Λ,Ξ) and the meson octet $(K,\pi,\eta,\bar{K})$, can be obtained from those of the boson self charge-conjugate octet by following relabeling of the particle names.

$$M_{I,Y}\rightarrow B_{I,Y}^{(20)}\,,$$

$$B_1\rightarrow Y_1^{(1)}\,,$$

$$B_2\rightarrow Y_1^{(2)}\,,$$

$$(K^*,\rho,\omega,\bar{K}^*)\rightarrow(N,\Sigma,\Lambda,\Xi)\,,$$

$$(K,\pi,\eta,\bar{K})\rightarrow(K,\pi,\eta,\bar{K})\,.$$

VI. SUM RULES FOR $27\rightarrow 8+8$ IN BROKEN SU(3)

We will denote the isospin and hypercharge multiplets of the 27 representation by (I,Y). The interest in the sum rules for this case is twofold:

(a) The 27 multiplet may be comprised of baryons [denoted by $B(27)$] decaying into $B(8)+P(8)$, $B(8)+V(8)$, or $B^*(8)+P(8)$, etc. Obviously, the result for this case would apply to any "27-plet" decaying into two *different* unitary octets.

(b) The 27-plet may be a meson multiplet, denoted by $M(27)$. The boson resonances classified in icosuplet by Lee, Okubo, and Schecter,[8] may equally well be fitted in the $M(27)$ and hence the interest in $M(27)$ decays into two octets. We assume $M(27)$ to be self-charge-conjugate, then the following particular cases of interest arise:

(i) $M(27)\rightarrow V(8)+P(8)$, i.e., decays into two distinct boson octets.

(ii) $M(27)\rightarrow P(8)+P(8)$, i.e., decays into two identical boson octets.

(iii) $M(27)\rightarrow B(8)+\bar{B}(8)$, i.e., the same baryon and antibaryon octets.

For completeness we consider all these cases. In the general case there are 24 coupling constants which are expressed in terms of seven parameters, one from exact SU(3) and six from first order "λ_8" breaking of SU(3), since the representation 27 occurs once in the decomposition of 8×8 and six times in $8\times8\times8$. Consequently, we expect 17 sum rules. We define the X_i's in terms of the coupling constants $G[(I,Y),BP]$ as follows:

$$X_1=G[(2,0),\Sigma\pi] =A-20a\,, \qquad (6.1)$$

$$X_2=(5)^{1/2}G[(1,0),N\bar{K}] =A+4a-2c-2d+3e-3f\,, \qquad (6.2)$$

$$X_3=(5)^{1/2}G[(1,0),\Xi K] =A+4a+2c+2d+3e+3f\,, \qquad (6.3)$$

$$X_4=(5)^{1/2}G[(1,0),\Sigma\pi] = +2c+2d-10f\,, \qquad (6.4)$$

$$X_5=(10/30^{1/2})G[(1,0),\Sigma\eta] =A+4a+2c-2d-2e\,, \qquad (6.5)$$

$$X_6=(10/30^{1/2})G[(1,0),\Lambda\pi] =A+4a-2c+2d-2e\,, \qquad (6.6)$$

$$X_7=(10/15^{1/2})G[(0,0),N\bar{K}] =A+16a -3e-15f, \quad (6.7)$$
$$X_8=-(10/15^{1/2})G[(0,0),\Xi K] =A+16a -3e+15f, \quad (6.8)$$
$$X_9=-(20/10^{1/2})G[(0,0),\Sigma\pi] =A+16a -18e, \quad (6.9)$$
$$X_{10}=[20/3(30)^{1/2}]G[(0,0),\Lambda\eta] =A+16a +2e, \quad (6.10)$$
$$X_{11}=\sqrt{2}G[(\tfrac{3}{2},1),N\pi] =A-5a-b-3c, \quad (6.11)$$
$$X_{12}=\sqrt{2}G[(\tfrac{3}{2},1),\Sigma K] =A-5a-b+3c, \quad (6.12)$$
$$X_{13}=(10/5^{1/2})G[(\tfrac{1}{2},1),N\pi] =A+13a-b +3d-9e-15f, \quad (6.13)$$
$$X_{14}=-(10/5^{1/2})G[(\tfrac{1}{2},1),\Sigma K] =A+13a-b -3d-9e+15f, \quad (6.14)$$
$$X_{15}=[10/3(5)^{1/2}]G[(\tfrac{1}{2},1),N\eta] =A+13a-b -d+e-5f, \quad (6.15)$$
$$X_{16}=[10/3(5)^{1/2}]G[(\tfrac{1}{2},1),\Lambda K] =A+13a-b +d+e+5f, \quad (6.16)$$
$$X_{17}=\sqrt{2}G[(\tfrac{3}{2},-1),\Xi\pi] =A-5a+b +3d, \quad (6.17)$$
$$X_{18}=\sqrt{2}G[(\tfrac{3}{2},-1),\Sigma\bar{K}] =A-5a+b -3d, \quad (6.18)$$
$$X_{19}=-(10/5^{1/2})G[(\tfrac{1}{2},-1),\Xi\pi] =A+13a+b-3c -9e+15f, \quad (6.19)$$
$$X_{20}=(10/5^{1/2})G[(\tfrac{1}{2},-1),\Sigma\bar{K}] =A+13a+b+3c -9e-15f, \quad (6.20)$$
$$X_{21}=[10/3(5)^{1/2}]G[(\tfrac{1}{2},-1),\Xi\eta] =A+13a+b+c +e+5f, \quad (6.21)$$
$$X_{22}=[10/3(5)^{1/2}]G[(\tfrac{1}{2},-1),\Lambda\bar{K}]=A+13a+b-c +e-5f, \quad (6.22)$$
$$X_{23}=G[(1,2),NK] =A+10a-2b, \quad (6.23)$$
$$X_{24}=G[(1,-2),\Xi\bar{K}] =A+10a+2b, \quad (6.24)$$

where A refers to the exact symmetry case, while a, b, c, d, e, and f are the amplitudes for $(SB')_{27_1}\to(BP)_{27}$, $(SB')_{27}\to(BP)_{27}$, $(SB')_{10}\to(BP)_{10}$, $(SB')_{10^*}\to(BP)_{10^*}$, $(SB')_8\to(BP)_{8_1}$, and $(SB')_8\to(BP)_{8_2}$, respectively. 'S' refers to the "spurion octet," and B' to the 27-plet.

Eliminating the seven parameters A, a, b, $\cdots$, f, we obtain the sum rules

$$X_1+X_{23} =X_{11}+X_{12}, \quad (6.25)$$
$$X_1+X_{24} =X_{17}+X_{18}, \quad (6.26)$$
$$X_{13}+3X_{15} =2X_7+2X_{23}, \quad (6.27)$$
$$X_{19}+3X_{21} =2X_8+2X_{24}, \quad (6.28)$$
$$X_6+3X_{10} =2X_{16}+2X_{22}, \quad (6.29)$$
$$X_5+3X_{10} =2X_{15}+2X_{21}, \quad (6.30)$$
$$X_{20}+3X_{22} =2X_7+2X_{24}, \quad (6.31)$$
$$X_{14}+3X_{16} =2X_8+2X_{23}, \quad (6.32)$$
$$3X_6+X_9 =(\tfrac{2}{3}X_{11}+\tfrac{4}{3}X_{13})+(\tfrac{2}{3}X_{17}+\tfrac{4}{3}X_{19}), \quad (6.33)$$
$$3X_6+X_1 =[(5/3)X_{11}+\tfrac{1}{3}X_{13}]+[(5/3)X_{17}+\tfrac{1}{3}X_{19}], \quad (6.34)$$
$$3X_5+X_9 =(\tfrac{2}{3}X_{12}+\tfrac{4}{3}X_{14})+(\tfrac{2}{3}X_{18}+\tfrac{4}{3}X_{20}), \quad (6.35)$$
$$3X_5+X_1 =[(5/3)X_{12}+\tfrac{1}{3}X_{14}]+[(5/3)X_{18}+\tfrac{1}{3}X_{20}], \quad (6.36)$$
$$X_9+X_{15}+X_{16} =X_{10}+X_{13}+X_{14}, \quad (6.37)$$
$$X_9+X_{21}+X_{22} =X_{10}+X_{19}+X_{20}, \quad (6.38)$$
$$X_8+X_{11}+X_{13}+X_{24} =X_7+X_{17}+X_{19}+X_{23}, \quad (6.39)$$
$$3X_2+2X_3+X_9 =2X_5+2X_6+X_{10}, \quad (6.40)$$
$$3X_4+X_8+X_{11}+X_{18} =X_7+X_{12}+X_{17}, \quad (6.41)$$
$$3X_2+(3/5)X_8+2X_{12}+2X_{17}=3X_3+(3/5)X_7+2X_{11}+2X_{18}. \quad (6.42)$$

It should be noted though that the ten sum rules (6.27) to (6.36) both inclusive have a linear relation between them and constitute in reality only nine linearly independent sum rules. However, we give the ten explicitly as they all have the same structure as the Gell-Mann–Okubo mass formula.

Case b(i): $M(27)\to V(8)+P(8)$. Since $M(27)$ is a self charge-conjugate boson multiplet, this requires that the nine equalities (or trivial sum rules) should hold, viz.,

$$\begin{aligned}&X_3=X_2, \quad X_8=X_7, \quad X_{17}=X_{11},\\ &X_{18}=X_{12}, \quad X_{19}=X_{13}, \quad X_{20}=X_{14},\\ &X_{21}=X_{15}, \quad X_{22}=X_{16}, \quad X_{24}=X_{23}.\end{aligned} \tag{6.43}$$

In terms of the parameters, these require $b=0$, $c=-d$, and $f=0$ in equations (6.1)–(6.24). Feeding in the equalities (6.43) in (6.25)–(6.42), we obtain the eleven independent sum rules for this particular case. Note, however, that the ten linearly dependent sum rules (6.27) to (6.36) will reduce to eight sum rules of which only seven are actually independent. The most striking sum rule which comes out in this case is $X_4=0$, i.e., $G[(1,0),\rho\pi]=0$.

Case b(ii): $M(27)\to P(8)+P(8)$. In this particular case Bose statistics require that $M(27)$ have $J^P 0^+$, 2^+, etc. The sum rules for this case may be obtained by feeding in the equalities

$$X_6=X_5, \quad X_{12}=X_{11}, \quad X_{14}=X_{13}, \quad X_{16}=X_{15}, \tag{6.44}$$

in addition to the equalities Eq. (6.43) for the case b(i). In terms of parameters this means that $b=c=d=f=0$ in this case. There will be eight independent sum rules, including $G[(1,0),\pi\pi]=0$.

Case b(iii): $M(27)\to B(8)+\bar{B}(8)$. For completeness we consider this case of coupling of a baryon octet to a self-conjugate boson $M(27)$. In this case we have $b=0$ and $c=d$ which immediately leads to the eight equalities

$$\begin{aligned}&X_6=X_5,\ X_{17}=X_{12},\ X_{18}=X_{11},\ X_{19}=X_{14},\\ &X_{20}=X_{13},\ X_{21}=X_{16},\ X_{22}=X_{15},\ \text{and}\ X_{24}=X_{23}.\end{aligned} \tag{6.45}$$

Since there are five parameters and eight equalities (6.45), we expect eleven sum rules. These may be obtained from the general case [Eqs. (6.25)–(6.42)] by feeding in the relations (6.45).

VII. DISCUSSION

Earlier we remarked that the vector plus pseudoscalar meson resonances may equally well form an $M(27)$ rather than an icosuplet. We make the identifications:

member of 27	resonance	mass (MeV)	width (MeV)
(2,0)	$(\rho\pi)$—(Ref. 9)	1200	350 MeV
(1,0)	$(\omega\pi)$—(Ref. 10)	1220	100 ± 20
$(\frac{3}{2},1)$ or $(\frac{1}{2},1)$	$(K^*\pi)$ or $(K\pi\pi)$—(Ref. 11)	~1230	80 MeV.

It should be emphasized that if the isospin of the $(\pi\rho)$ resonance is found to have $I=2$, then it cannot be accommodated in an octet or an icosuplet and the 27-plet is then the only choice. But, if the $(\pi\rho)$ resonance is found to have $I=1$, then it can be put only in an octet or an icosuplet, because the (1,0) member of the 27-plet does not decay into $\pi+\rho$ even when symmetry breaking to first order is taken into account. Consequently, we take the $(\pi\rho)$ resonance to be (2,0) and the $(\pi\omega)$ resonance to be the (1,0) members of the 27-plet. With two input masses, namely those of the $(\pi\omega)$ resonance and the $(K\pi\pi)$ resonance, one can predict the masses of all the other members of the 27-plet. All these come out to be in the neighborhood of 1200 to 1250 MeV whether one chooses $(K\pi\pi)$ to be $(\frac{1}{2},1)$ or $(\frac{3}{2},1)$. Assuming then that the masses of the multiplets in the $M(27)$ lie in the range 1200 to 1300 MeV, we note that the coupling constants related with X_1, X_4, X_6, X_9, X_{11}, and X_{13} would be measurable directly from the strong interaction decay of the resonance. Thus, in case b(i), three sum rules would be amenable to experimental check, viz.,

$$3X_6+X_9=\tfrac{4}{3}X_{11}+(8/3)X_{13}, \tag{7.1}$$

$$X_1+3X_6=(10/3)X_{11}+\tfrac{2}{3}X_{13}, \tag{7.2}$$

and

$$X_4=0. \tag{7.3}$$

At present the first two cannot be tested as X_9 is not known and only one of the X_4 and X_{13} is known. The fact that the B particle or the $(\omega\pi)$ resonance does not decay into $\rho+\pi$, i.e., $X_4=0$, is also expected to hold because of G invariance and cannot be taken as evidence in favor of our sum rule.

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